SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and Prove Lebesgue dominated convergence theorem.

17. State and Prove the Riesz-Fischer theorem.

18. State and Prove Jordan Theorem.

19. If both partial derivatives $D_r f$ and $D_k f$ exist in an n – *ball* $B(c)$ and if both $D_{r,k} f$ and $D_{k,r} f$ are continuous at c, prove that $D_{r,k} f(c) = D_{k,r} f(c)$.

20. State and Prove Inverse function theorem.

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APRIL/MAY 2024

GMA22 — REAL ANALYSIS — II

Shanmuga Industries Arts & Science College T.V.Malai

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Define monotonic sequences of step function.

2. State Levi theorem for sequences of Lebesgue-integrable functions.

3. If $f(x) = \frac{1}{(1+x^2)}$ for all x in R, Prove that $f \in L(R)$.

4. If $f \in L(I)$ and if f is bounded almost everywhere on I, prove that $f^2 \in L(I)$.

5. Define Orthogonal and orthonormal on I.

6. State Dini's test.

7. Write Cauchy – Schwarz inequality.

8. Define Directional Derivative.

9. State Implicit function theorem

10. State Second – derivative test for extrema.

SECTION B — (5 × 5 = 25 marks)

Answer ALL the questions.

11. (a) State and Prove Levi theorem for upper function.

Or

(b) Let $f(x)=x^s$ for $x>0$, $f(0)=0$. Prove that the Lebesgue integral $\int_0^1 f(x)\,dx$ exists and has the value $\frac{1}{(s+1)}$ if $s>-1$.

12. (a) If f be defined on I and assume that $\{f_n\}$ is a sequence of measurable functions on I such that $f_n(x)\to f(x)$ almost everywhere on I, prove that f is measurable on I.

Or

(b) If a complex valued function f is Lebesgue- integrable on I, prove that $|f|\in L(I)$ and $\left|\int_I f\right|\le\int_I |f|$.

13. (a) State and Prove Weierstrass approximation theorem.

Or

(b) Let $\{\varphi_0,\varphi_1,\ldots\ldots\}$ be Orthonormal on I and $\{c_n\}$ be any sequence of complex numbers such that $\sum|c_n|^2$ converges , prove that there is a function f in $L^2(I)$ and $\|f\|^2=\sum_{k=0}^{\infty}|c_k|^2$.

14. (a) State and Prove Taylor's formula.

Or

(b) State and Prove Mean value theorem for differentiable function.

15. (a) Let A be an open subset of R^n and assume that $f:A\to R^n$ has continuous partial derivatives $D_j f_i$ on. If $J_f(x)\neq 0$ for all $\in A$. Prove that f is an open mapping.

Or

(b) A quadric surface with centre at the origin has the equation $Ax^2+By^2+Cz^2+2Dyz+2Ezx+2Fxy=1$. Find the lengths of its semi – axes.